# Inflation and New Agegraphic Dark Energy

Cheng-Yi Sun*,1 and Rui-Hong Yue†,2

[1]Institute of Modern Physics, Northwest University, Xian 710069, P.R. China.

[2]Faculty of Science, Ningbo University, Ningbo 315211, P.R. China.

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**Abstract**

In the note, we extend the discussion of the new agegraphic dark energy (NADE) model to include the inflation stage. Usually, in the inflation models, for convenience the conformal time $\eta$ is set to be zero at the end of inflation. This is incompatible with the NADE model since $\eta = 0$ indicates the divergence of NADE. To avoid the difficulty, we can redefine the conformal time as $\eta + \delta$. However, we find that the positive constant $\delta$ must be so large that NADE can not become dominated at present time.

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**Key words:** agegraphic dark energy, conformal time, inflation

Increasing evidence suggests that the expansion of our universe is being accelerated [1, 2, 3]. Within the framework of the general relativity, the acceleration can be phenomenally attributed to the existence of a mysterious exotic component with negative pressure, namely the dark energy [4, 5]. However, we know little about the nature of dark energy. The most nature, simple and important candidate for dark energy is the Einstein's cosmological constant, which can fit the observations well so far. But the cosmological constant is plagued with the well-known fine-tuning and cosmic coincidence problems [4, 5]. Dark energy has become one of the most active fields in the modern cosmology.

Recently, a new dynamical model of dark energy, the new agegraphic dark energy (NADE) model, is proposed [6]. The energy density of agegraphic dark energy is proposed to be [6]

$$\rho_q = \frac{3n^2 M_p^2}{\eta^2}. \tag{1}$$

*cysun@mailis.gucas.ac.cn; ddscy@163.com

†yueruihong@nbu.edu.cn

Here $n$ is a constant pramameter, $M_p = (8\pi G)^{-1/2}$ and $\eta$ is the conformal age of the universe

$$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da}{Ha^2}, \tag{2}$$

where $a$ is the scale factor, $H \equiv \dot{a}/a$ is the Hubble parameter and a dot denotes the derivative with respect to the cosmic time $t$.

The NADE model is successful in explaining the accelerated expansion and in fitting the observational data [7, 8]. The evolution of NADE has been surveyed in detail. It has been shown that NADE becomes dominated in the future, and negligible in the matter-dominated epoch and in the radiation-dominated epoch, as expected [6].

Since it is generally believed that before the radiation-dominated epoch there exists another important stage, *Inflation*, it is natural for us to extend the discussion of the NADE model to include the inflation stage. It seems that only trivial results would be obtained since NADE should be negligible in the inflation stage. However, we find that there exists incompatibilities between the inflation scenario and the NADE model.

Let us show it. From Eq.(1), we know that once the conformal time is given, $\rho_\eta$ will be determined. It is known that in the inflation models the conformal time $\eta$ is an important parameter which is widely used in calculating the primordial spectrum of the curvature perturbation, and usually is defined as

$$d\eta = \frac{dt}{a} = \frac{da}{Ha^2}. \tag{3}$$

Since in a slow-rolling inflation model, $H$ is constant approximately, then by integrating Eq.(3), we can get

$$\eta - \eta_i \simeq \frac{1}{H}\Big(\frac{1}{a_i} - \frac{1}{a}\Big) \simeq -\frac{1}{Ha} - \Big(-\frac{1}{H_i a_i}\Big), \tag{4}$$

where the subscript $i$ denotes the value of the corresponding parameter at some initial moment during the inflation stage. Then by choosing $\eta_i = -\frac{1}{H_i a_i}$, we can get a well-known result

$$\eta \simeq -\frac{1}{Ha}. \tag{5}$$

Of course, we can express $\eta$ more precisely by using the slow-rolling parameters. Here what is important is not the precise form but the fact that $\eta$ is *negative* in the inflation scenario. In fact, it is well known that, during the inflation stage,

$$-\infty < \eta < 0,$$

and usually, in the literature it is set

$$\eta_E = 0$$

at the end of slow-roll inflation for convenience [9]. Hereafter the subscript $E$ denotes the value of the corresponding parameter at the end of the inflation. On the other hand, in the radiation-dominated epoch, with $\eta_E = 0$ and Eq.(2) we get

$$\eta = \frac{1}{Ha} > 0. \tag{6}$$

Then the fact that $\eta$ is negative during the inflation stage and positive in the radiation-dominated epoch indicates that $\eta$ must be zero at some moment before the radiation-dominated epoch. Obviously, this is unacceptable since $\eta = 0$ indicates that $\rho_q$ defined in Eq.(1) is infinite. So Eq.(5) that is widely used in the inflation models contradicts the NADE model.

It seems that the contradiction can be removed easily if we do not choose $\eta_i = -\frac{1}{a_i H_i}$. Then we can define a positive conformal time from Eq.(4) as

$$\eta \simeq \delta - \frac{1}{Ha}, \tag{7}$$

where $\delta = \eta_i + \frac{1}{a_i H_i}$ is a constant parameter. To remove the zero point of $\eta$, we should require that

$$\delta > \frac{1}{Ha}$$

holds during the inflation stage. This can be guaranteed by the requirement

$$\delta \geq \frac{1}{H_I a_I}, \tag{8}$$

since $Ha$ is increasing in the inflation stage. Hereafter the subscript $I$ denotes the value of the corresponding parameter at the beginning of the inflation. Then at the end of the inflation, we have

$$\eta_E = \delta - \frac{1}{a_E H_E} \geq \frac{1}{a_I H_I} - \frac{1}{a_E H_E}. \tag{9}$$

In fact, the definition of Eq.(5) is to set $\eta_E = 0$, while Eq.(7) is to set

$$\eta_I \geq 0.$$

Then, with Eq.(7), the contradiction between inflation and NADE displayed in the last paragraph is eliminated.

However, the other problem results from Eq.(7). From Eqs.(2), after the end of the inflation, we have

$$\eta = \eta_E + \int_{t_E}^{t} \frac{dt'}{a(t')}. \tag{10}$$

Obviously, at the present time, we have

$$\eta_0 > \eta_E \geq \frac{1}{a_I H_I} - \frac{1}{a_E H_E}, \tag{11}$$

where Eq.(9) has been used. Hereafter the subscript 0 denotes the value of the corresponding parameter at the present time. Then by defining $\Omega_q = \frac{n^2}{H^2\eta^2}$, we have

$$\Omega_{q0} < \frac{n^2}{H_0^2} a_I^2 H_I^2 \Big(1 - \frac{a_I H_I}{a_E H_E}\Big)^{-2}. \tag{12}$$

We define the comoving wavenumber $k_*$ corresponding to the present Hubble scale as

$$k_* = H_0 a_0. \tag{13}$$

Then to solve the horizon problem, we should require

$$k_* > H_I a_I.$$

In fact, the requirement is not enough. Generally, in order for the cosmological perturbation on the Hubble scale to be generated from the quantum fluctuation, we should require

$$k_* \gg H_I a_I. \tag{14}$$

Then together with Eq.(13), we have

$$\frac{H_I^2 a_I^2}{H_0^2 a_0^2} \ll 1 \tag{15}$$

By analyzing the observational data, it has been shown in [8] that with the choice of $a_0 = 1$, $n = 2.886^{+0.084}_{-0.082}$ at $1\sigma$ confidence level, and $n = 2.886^{+0.169}_{-0.163}$ at $2\sigma$ confidence level. Then from Eq.(15), we know

$$n^2 \frac{H_I^2 a_I^2}{H_0^2 a_0^2} \ll 1. \tag{16}$$

With the choice of $a_0 = 1$, we may rewrite Eq.(12) as

$$\Omega_{q0} < n^2 \frac{a_I^2 H_I^2}{a_0^2 H_0^2} \Big(1 - \frac{a_I H_I}{a_E H_E}\Big)^{-2}. \tag{17}$$

At the same time, we know

$$a_I H_I \ll a_E H_E.$$

Then, from Eq.(17), we can conclude

$$\Omega_{q0} \ll 1. \tag{18}$$

This is also unacceptable. Then, using Eq.(7), the zero point of $\eta$ is removed and the divergence of $\rho_q$ is avoided, but we find that with Eq.(7), NADE can not become dominated at the present time.

To summary, in the note, we show that Eq.(5) that is widely adopted in the inflation models implies the existence of the zero point of $\eta$ which subsequently indicates the divergence of $\rho_q$. So Eq.(5) is incompatible with the NADE models. It seems that the incompatibility can be removed easily if Eq.(5) is replaced by Eq.(7). However, we find that Eq.(7) indicates that $\Omega_{q0} \ll 1$. Then we think that the inflation scenario may be incompatible with the NADE model. Here we note that the inflation scenario and the NADE model are very successful in their own fields. Although it has been shown that there exists the contradiction between them, we do not think that either the inflation scenario or the NADE model is incorrect. There may exists some unknown way out of the incompatibility.

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